The Use of Wiener Deconvolution (an Optimal Filter) in Nuclear Magnetic Resonance Imaging

John H. Letcher

Department of Mathematical and Computer Sciences, University of Tulsa,Tulsa, Oklahoma 74104

A computer software technique has been developed and employed to insert a modified-Wiener-deconvolution filter (an optimal filter) immediately following the quadrature detector of a magnetic resonance imager. The purpose of this effort is to try to suppress the undesirable effect of the noise found in the magnetic resonance imaging receiving antenna. This technique convolves the sampled real and imaginary signals taken directly from the antenna quadrature detector with a transfer function which is expressed in terms of the power spectrum of the measured signal with noise and the measured power spectrum of the noise, alone. The latter factor is measured during the dummy cycles which precede a normal scan sequence. Results are presented using data taken with a commercially available phantom.

INTRODUCTION

Nuclear Magnetic Resonance Imaging (MRI) is a good example of a computer-driven experiment. Through the use of an equation describing rather well-understood physical phenomena, it has been possible to design the conditions under which images, i.e., density maps, of living tissue can be generated, noninvasively.

Within the MRI equipment, a receiving antenna detects signals emitted by the patient that are stimulated by a sequence of radio frequency pulses while the patient is immersed in a magnetic field. These signals, from which images are calculated, are received as a sequence of complex time series of voltages from the receiving antenna. Unfortunately, the accuracy of this procedure is marred by the presence of additive noise generated in our about the receiving antenna. Such noise can be static or power surging in the electrical supply, strong local RF sources, such as radio/television transmitters and other adjacent, powerful medical equipment, or noise generated within the MRI itself. The reduction in image quality is evidenced by shadows, haze, and/or bands of horizontal lines. Such reduction in image quality can present a problem to a medical practitioner trying to correctly detect abnormal tissue within a patient.

To ensure that such noise is reduced or eliminated, the MRI is equipped with a shielded filtered power supply and a plurality of RF shields. Further, very careful attention is paid to proper manufacturing of components and proper maintenance. Due to continued use and the inherent complexity of the MRI, the undesirable effects of such received noise often becomes apparent. Usually, the MRI is then shut down until a repair person can adjust the RF shields, etc., to try to correct the problem. However, ceasing the use of the MRI is wasteful, and even the most skilled technician cannot remove noise caused by the MRI's internal components.

There is a need for a simple system that can be used on existing machines as well as incorporated into the manufacture of an MRI to suppress or eliminate the undesirable effects of noise. This paper describes the use of one optimal filtering technique in the MRI image reconstruction process which overcomes the foregoing deficiencies and meets the above described needs.

Since the raw data samples are not usually available to the researcher in a commercial imager, equipment was designed and built to replace the entire receiving end of a commerical MRI. This equipment was used to implement the filtering technique described herein.

The MRI experiment is well understood and is described adequately elsewhere [1, 2]. However, a brief review is given below to define terms which are used later in this paper.

THE MRI EXPERIMENT

By applying a properly designed RF pulse to a sample, we can induce conditions such that the observed signal from a small volume in a magnetic field $\mathbf{H}_0$ is expressed by

$$dS = dx\, dy\, dz\, N(x, y, z) \exp(-2\pi i f t + \phi) \qquad (1)$$

where N is proportional to the number of spins within this infinitesimal volume at coordinates (x, y, z). These spins are the intrinsic angular momentum (magnetic moment) of the hydrogen nuclei (protons) within the body. The frequency f of the emitted radiation is directly proportional to the imposed magnetic field strength $\mathbf{H}$. Therefore, by superimposing gradients in two of the orthogonal directions on the steady magnetic field, it is possible to (1) select for excitation only spins within one plane and (2) cause a distribution in emitted frequencies in a plane perpendicular to the selected plane.

The experiment proceeds as follows. A sequence of RF pulses and imposed gradients are used so that the signal may be sampled. This produces a *time series* of a complex signal. One then generates a sequence of these time series, one different from the other, by varying a small magnetic field gradient in the third orthogonal direction. This generates a pseudo time frequency so that looking at the set of time series (t_1, t_2) as a whole the received signal is expressed as

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$$g(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(f, \Phi) e^{-2\pi i (f t_1 + \Phi t_2)}\, df\, d\Phi \qquad (2)$$

The gradients are selected so that $f = f_0 + x$ and $\Phi = f_0 + y$, where f_0 is the Larmor frequency, at which resonance is produced by $\mathbf{H}_0$.

The sampled data in the MRI experiment are measurements of $g(t_1, t_2)$. For example, in a 256 by 256 image, 256 sequences of 256 equally spaced sampled points of a complex signal are measured by two-channel analog-to-digital conversions. By two-dimensional fourier transformation, the spin density function $G(x, y)$ is given by

$$G(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t_1, t_2) e^{2\pi i (x t_1 + y t_2)}\, dt_1\, dt_2 \qquad (3)$$

Since the signal is sampled N times, the continuous function $g(t_1, t_2)$ is recorded as a complex array $g(pT_x, qT_y)$, where p and q are integers in the interval [0, 255], and T_x and T_y are constants for a given experiment.

Using $T_x = T_y = 4 \times 10^{-5}$ s for the examples below and $N = 256$, and if n and m are integers in the interval [0, 255], then

$$G(n/NT_x, m/NT_y) = \sum_{q=0}^{N-1} \sum_{p=0}^{N-1} g(pT_x, qT_y) e^{2\pi i n p/N} e^{2\pi i m q/N} \qquad (4)$$

Notice that G and g are complex and are stored in the computer as complex arrays $\mathbf{G}(n, m)$ and $\mathbf{g}(p, q)$. The magnitude image $\mathbf{I}(n, m)$ is a real array calculated as follows:

$$\mathbf{I}(n, m) := (|\mathrm{Real}(\mathbf{G}(n, m))|^2 + |\mathrm{Imag}(\mathbf{G}(n, m))|^2)^{1/2} \qquad (5)$$

By the form of Eq. (2) and by the use of properties of the received time series (e.g., each of these is a wavelet), we may then apply two-dimensional fourier transformation to obtain the two-dimensional array of complex image data, the magnitude of which is the "image". Image reconstruction algorithms are derived by the forms of the equations and the properties of the functions, not by applying any physical laws. Theoretically, this procedure can be carried out to produce images of living tissue of extraordinary quality. There appears to be no theoretical limitation to the accuracy of this method. Unfortunately, additive noise is present in the received signal. Therefore, the two-dimensional fourier transformation is performed on the signal *plus noise* which produces a variety of artifacts, such as ghosts, haze, or lines, on the images. Fortunately, many workers [3, 4] have done work with time series of this sort that enable us, through computational techniques, to attenuate the effects of this noise.

THE EXPERIMENTAL EQUIPMENT

Equipment was designed and assembled to carry out the signal acquisition, filtering, fourier transformation, image display, image manipulation, and other tasks which was independent of, but worked in parallel with, the equipment included with the MRI.

The MRI gradients and RF signals were generated by a Picker International Vista MRI, and the magnet used was an Oxford 0.5 tesla superconducting magnet, which is in current use for medical purposes. The receiving coil could be a commercially supplied body coil or head coil or surface coils which have been designed by this author [5]. The receiving unit included a preamp/filter and quadrature detector to accept the signal from the MRI receiving antenna. A unit was built that included a dual channel 16-bit analog-to-digital conversion processor (an Analogic Corporation Shad 2). The control circuitry consisted of two finite state engines [6], which are small, extremely fast control processors which were adapted for this purpose. These engines were contained in a small enclosure that was placed next to the quadrature detector of the MRI. A fiber optic cable or ordinary flat cable (depending upon the distance to the receiving computer) sent the real and imaginary 16-bit samples (taken at a rate of 100,000 samples per second) to a specially built circuit card. This circuit card was inserted into an industrial grade IBM PC/AT-type computer (a Diversified Technology CAT 901) where the memory bus and I/O bus were run at 10 MHz with zero wait states. The image reconstruction and filtering described below were performed using the Intel 80286 and 80287 processors with software written by the author. The fourier transformation software was a slight variant of the Cooley Tukey algorithm [7]. Image display was accomplished using Matrox professional image processor (PIP 640B) which fed an Electrohome 15 inch 1000 line resolution 256 level gray scale monitor.

OPTIMAL FILTERING

In the MRI experiment, the observed complex signal $\mathbf{s}(t_1, t_2)$ is not clean. In the data collection process, the actual signal $\mathbf{g}(t_1, t_2)$ has been mixed with noise $\mathbf{n}$ such that

$$\mathbf{s}(t_1, t_2) = \mathbf{g}(t_1, t_2) + \mathbf{n}(t_1, t_2)\,. \qquad (6)$$

Let's assume that $\mathbf{g}$ and $\mathbf{n}$ are quantities whose power spectra are known (in principle at least, when sufficient data are available to achieve the desired accuracy). In the MRI experiment, the noise to be suppressed is uncorrelated with respect to the signal. The power spectrum of the noise (the signal plus noise recorded without signal) is measurable as dummy cycles before the actual scan is initiated. In most instances, the noise is small with respect to the magnitude of the signal, but need not be so.

The design of the digital filter is such that the observed signal $\mathbf{s}$ is transformed into a signal $\mathbf{y}$ which, in the least square error sense, is as close to $\mathbf{g}$ as possible. For $\mathbf{y}$ to equal $\mathbf{g}$ is simply too much to ask for any linear filter. The process for calculating the transfer function for this filter is outlined elsewhere for the case when the signal without noise is measurable [5]. These data are not available, but it can be asserted that the power spectrum of the signal without noise is that of the signal with noise minus the power spectrum of the noise. Therefore, the proposed transfer function is $\mathbf{h}_j = (\mathbf{P}_{sj} - \mathbf{P}_n)/\mathbf{P}_{sj}$. The latter term serves to normalize $\mathbf{h}$ to unity when $\mathbf{P}_n = 0$.

The transfer function $\mathbf{h}$, when convolved with the observed signal $\mathbf{s} = \mathbf{g} + \mathbf{n}$, produces a good approximation of the actual signal $\mathbf{g}$. The process is as follows:

1. Obtain sample data sets $\mathbf{n}$ for ambient noise, alone.
2. Obtain a data set of the observed signal $\mathbf{s}_j$, $j = 1, \ldots, 256$.

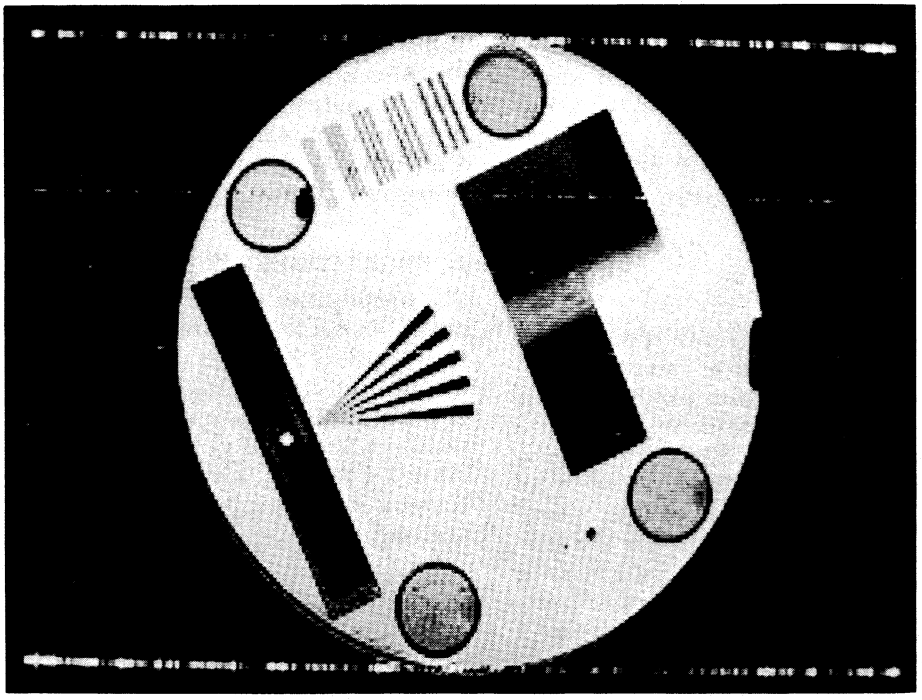

Figure 1. The magnitude image of the phantom calculated without any attempt to remove the noise in the received signal.

3. Calculate the fourier transform of the autocorrelation of the signal of noise, alone $\equiv \mathbf{P}_n$.
4. Calculate the fourier transform of the autocorrelation function of the signal with noise, $\equiv \mathbf{P}_{sj}$.
5. Calculate the transfer function $\mathbf{h}_j = (\mathbf{P}_{sj} - \mathbf{P}_n)/\mathbf{P}_{sj}$ for each data set.
6. Obtain the convolution of the transfer function $\mathbf{h}_j$, with the observed signal $\mathbf{s}_j$, yielding $\mathbf{g}_j$. Note that the transfer functions $\mathbf{h}_j$ are different for each of the data sets $\mathbf{s}_j$.

Since a fourier transformation is to be performed on each $\mathbf{g}_j$ as a part of the image reconstruction process, the process of

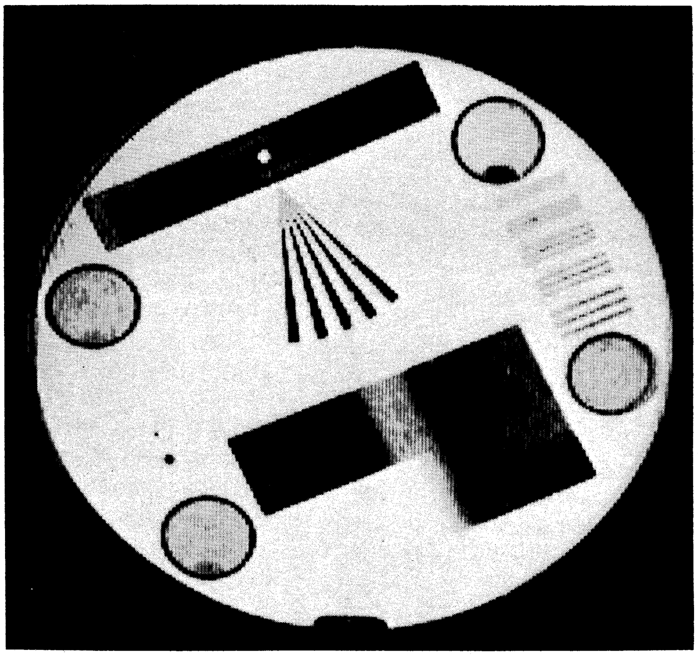

Figure 2. The magnitude image of the phantom calculated using the calculated deconvolution filter described in the text.

convolving $\mathbf{h}_j$ with $\mathbf{s}_j$ is never carried out. Using straightforward programming techniques, the cost of performing this filtering technique is the price of the calculation of an autocorrelation (which is equivalent to another fourier transformation plus a multiplication). Therefore, in a normal reconstruction of a 256 by 256 image, 2×256 fourier transformations are calculated. Using this technique, 3×256 are required.

STUDIES WITH A PHANTOM

A "phantom" is a large block of plexiglas into which is inserted geometric objects of different magnetic permeabilities which is used to test the accuracy of the MRI. A phantom manufactured by Picker International was used to provide experimental data. The above described equipment was run and was tampered with to induce a variety of different types of noise to test the performance of the optimal noise filtering. For example, in one test a grounding lead from the imaging unit was intentionally broken, thereby inducing additive noise to the imaginary signal, and a coherent essentially single frequency signal was also induced to produce a number of artifacts such as haze and broad noise lines that greatly diminish the interpretibility of the image. Thereafter, the same data with added noise were filtered by the technique described herein. Figures 1 and 2 show the results before and after the filtering process. Notice a significant improvement in the quality of the image by the elimination of the haze and horizontal lines *without* any apparent degradation in the unaffected parts of the image itself.

There are two instances when this filtering technique will not improve the quality of the images. The first is when the noise is white, i.e., when $\mathbf{P}_n$ is a constant. The second is when the power spectrum of the noise is identical to that of the signal. That is when $\mathbf{P}_n \equiv \mathbf{P}_{sj}$, then $\mathbf{h}_j$ is a constant, a no filter. Uusualy, this is not the case.

Even though the use of this process adds the additional computational burden which is the equivalent of 256 fourier transforms, there is normally enough computer time available *during data collection* to carry out this process if a fast algorithm is used. Adding this step should not delay the time spent in producing an image. At the very least, the calculated $\mathbf{P}_n$ is a monitor of the operating condition of the MRI unit. It is felt that this procedure can be successfully used in the image reconstruction process in essentially all images.

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