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● *Original Contribution*

COMPUTER-ASSISTED DESIGN OF SURFACE COILS USED IN MAGNETIC RESONANCE IMAGING. I. THE CALCULATION OF THE MAGNETIC FIELD

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For a number of reasons, it is desirable to fabricate coils which, for a known current, shall produce predetermined values of the magnetic field intensity at a number of points within a nuclear magnetic resonance imager. The calculation of the magnetic field intensity at a set of points involves the integration of the Biot–Savart equation for all components of the segments of conductor which make up the coil. This process in itself is a rather formidable task. When this process is parameterized in terms of coil diameter, coil spacing, etc. the problem is to determine the values of these parameters to match values of magnetic field intensities which are desired. The problem thereby increases in complexity to the point where, by ordinary methods, the problem becomes intractable. This note describes an algorithm and offers a computer subroutine to calculate magnetic fields for coils of arbitrary shape and complexity for fixed currents.

Keywords: **MRI surface coils.**

INTRODUCTION

In the use of nuclear magnetic resonance imaging, it is sometimes desirable to have and use coils other than the whole body and head units customarily supplied by the manufacturer of the imager. Coils are also needed which are specific to the study of (1) the lumbar spine, (2) the thoracic spine, (3) the knee and shoulder, and (4) one to properly study specimens immediately after removal by surgical techniques. In the design of a new coil, the magnetic field distribution is specified for the desired unit, properly scaled to suit the dimensions and characteristics of the specimen under investigation. The purpose of the study undertaken herein is to produce a technique allowing the calculation of the magnetic field intensities at specific points in space. These will be used in the studies to determine the shape and dimensions of the coil which, once fabricated, will produce the desired magnetic field distribution. The calculated magnetic field should give a direct measure of the sensitivity of the coil when the coil is used as an antenna in a magnetic resonance imager.

The magnetic field produced by a current moving in a collection of line segments of wire is, in theory at least, calculable.[1] That is, by integration of the Biot–Savart equation, it is possible to calculate the magnetic field intensity throughout the experimental volume.

THE BIOT–SAVART EQUATION

The total magnetic field $\hat{B}$ produced at a point at a distance R from a current I flowing in a wire configuration is given by:

$$\hat{B} = \frac{\mu_0 I}{4\pi} \int_c \frac{(d\hat{L} \times \hat{r})}{|r|^3} \quad , \tag{1}$$

where $\hat{r}$ is the vector between the infinitesimal conductor line element $d\hat{L}$ and the observation point $\hat{P}$ and $\mu_0(= 4 \times 10^{-7})$ is the permeability of free space.

In special cases, e.g. at the center of a circular conducting loop, the above equation is readily integrated. However, for even modestly complex shapes, and for instances where the magnetic field intensity is to be de-

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termined off axis, the integrals become quite difficult or impossible to evaluate in closed form.

It is possible for the above integral for Eq. (1) to be integrated for arbitrary shapes using numerical techniques on a digital computer but to resort to this technique one faces the fact that the calculational process will be quite slow. Going on the assumption that the shapes to be employed are expressible in terms of functions which generate integrals which can be integrated in closed form, let us consider the situation wherein each geometric shape of current carrying conductor is expressible in terms of single-parametric equations. If this technique is successful, integrals obtained thereby will normally be a function of one variable for the calculation of the three vector components of the magnetic field.

In general, it is desirable to express the equation of a line in terms of a single parameter t, as follows:

$$\hat{L} = \hat{L}(t) = \hat{i}\,x(t) + \hat{j}\,y(t) + \hat{k}\,z(t) \ . \qquad (2)$$

The infinitesimal segment $d\hat{L}$ is therefore given by the following equation:

$$d\hat{L} = \frac{d\hat{L}(t)}{dt}\,dt = \left[\hat{i}\,\frac{dx(t)}{dt} + \hat{j}\,\frac{dy(t)}{dt} + \hat{k}\,\frac{dz(t)}{dt}\right]dt \ . \qquad (3)$$

In terms of the above, the vector $\hat{r}$ is thereby given by the following:

$$\hat{r} = \hat{i}(x(t) - x_0) + \hat{j}(y(t) - y_0) + \hat{k}(z(t) - z_0) \ . \qquad (4)$$

Let us now consider how the equations would be set up to evaluate the magnetic field of a straight line segment extending between coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2), Furthermore, let us say that it is our intent to evaluate the magnetic field intensity at a point (x_0, y_0, z_0). For the purposes of this example, (x_0, y_0, z_0) is not contained on the line extending between (x_1, y_1, z_1) and (x_2, y_2, z_2). The equation expressing the line segment in terms of a single parameter t is given by

```
      SUBROUTINE LINE(BX,BY,BZ,X0,Y0,Z0,X1,Y1,Z1,X2,Y2,Z2)
      IMPLICIT REAL*8(A-H,O-Z)
      RK=1.0D-7
      FOUR=4.0D0
      TWO=2.0D0
      DX01=X0-X1
      DY01=Y0-Y1
      DZ01=Z0-Z1
      DX21=X2-X1
      DY21=Y2-Y1
      DZ21=Z2-Z1
      A=(DX01**2)+(DY01**2)+(DZ01**2)
      B=(-TWO)*(DX01*DX21+DY01*DY21+DZ01*DZ21)
      C=DX21**2+DY21**2+DZ21**2
      DBX=DY21*DZ01-DY01*DZ21
      DBY=DX01*DZ21-DX21*DZ01
      DBZ=DX21*DY01-DX01*DY21
      WD=(FOUR*A*C*SQRT(A+B+C)-(B**2)*SQRT(A+B+C))
     1 *(FOUR*C*A**(1.5)-SQRT(A)*B**2)
      WN=(-TWO*B*SQRT(A+B+C)*(FOUR*A*C-B**2)+(TWO*B+FOUR*C)
     1 *(FOUR*C*A**(1.5)-SQRT(A)*B**2))
      T=RK*WN/WD
      BX=DBX*T
      BY=DBY*T
      BZ=DBZ*T
      RETURN
      END
```

Fig. 1. A FORTRAN language subroutine that calculates the contribution to the magnetic field (BX,BY,BZ) at a point (X0,Y0,Z0) for a unit current flowing from (X1,Y1,Z1) directly to (X2,Y2,Z2).

$$\hat{L}(t) = \hat{i}(x_1 + (x_2 - x_1)t) + \hat{j}(y_1 + (y_2 - y_1)t) + \hat{k}(z_1 + (z_2 - z_1)t) \quad (5)$$

$$d\hat{L} = [\hat{i}(x_2 - x_1) + \hat{j}(y_2 - y_1) + \hat{k}(z_2 - z_1)]\,dt \quad (6)$$

and the vector $\hat{r}$ is likewise given in terms of the same parameter t by the following:

$$\hat{r}(t) = \hat{i}[(x_1 - x_0) + (x_2 - x_1)t] + \hat{j}[(y_1 - y_0) + (y_2 - y_1)t] + \hat{k}[(z_0 - z_1) + (z_2 - z_1)t] \ . \quad (7)$$

In terms of the above, the numerator $d\hat{L} \times \hat{r}$ which is given by

$$d\hat{L} \times \hat{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (x_2 - x_1)dt & (y_2 - y_1)dt & (z_2 - z_1) \\ (x_1 - x_0) + (x_2 - x_1)t & (y_1 - y_0) + (y_2 - y_1)t & (z_1 - z_0) + (z_2 - z_1)t \end{vmatrix} \quad (8)$$

and the demoninator $|r|^3$ is

$$|r|^3 = \{[(x_1 - x_0) + x_2 - x_1)t]^2 + [(y_1 - y_0) + (y_2 - y_1)t]^2 + [(z_1 - z_0) + (z_2 - z_1)t]^2\}^{3/2} , \quad (9)$$

thereby producing a relatively simple equation of the form:

$$\frac{d\hat{L} \times \hat{r}}{|r|^3} = \hat{i}\int \frac{D_x dt}{(A + Bt + Ct^2)^{3/2}} + \hat{j}\int \frac{D_y dt}{(A + Bt + Ct^2)^{3/2}} + \hat{k}\int \frac{D_z dt}{(A + Bt + Ct^2)^{3/2}},$$

where

$$A = (x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2$$
$$B = 2[(x_1 - x_0)(x_2 - x_1) + (y_1 - y_0)(y_2 - y_1) + (z_1 - z_0)(z_2 - z_1)]$$
$$C = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$
$$D_x = (y_2 - y_1)(z_1 - z_0) - (z_2 - z_1)(y_1 - y_0)$$
$$D_y = (z_2 - z_1)(x_1 - x_0) - (x_2 - x_1)(z_1 - z_0)$$
$$D_z = (x_2 - x_1)(y_1 - y_0) - (y_2 - y_1)(x_1 - x_0) \ . \quad (10)$$

To perform the integral along this line segment, the parameter t is integrated between the limits of 0 and 1 as dictated by the definition of the parametric equation for the straight line segment.

Figure 1 is a FORTRAN subroutine that calculates the contribution of the magnetic field, (BX,BY,BZ) at a point (X0,Y0,Z0) for a unit current flowing from a point (X1,Y1,Z1) to (X2,Y2,Z2). The magnetic field produced by a sequence of straight line segments is simply the sum of the fields which are calculated for each line segment. Therefore, through multiple calls to the subroutine listed in Fig. 1 the three components of the magnetic field of a coil of arbitrary shape can be calculated.

REFERENCES

1. Mansfield, P.; Morris, P.G. NMR Imaging in BioMedicine. San Diego, CA: Academic Press; 1982.